Form No. T611

Philadelphia University
Faculty of Engineering

Student Name:
Student Number:

Dept. of Electrical Engineering
First Exam, Second Semester: 2018/2019

Course Title: Electromagnetics I	**Date: 24/3/2019**
Course No: (610213)	**Time Allowed: 50 Minutes**
Lecturer: Dr. Mohammad Abu-Naser	**No. of Pages: 3**

Question 1: ***(35Mark)***

Objectives: This question is related to Coulomb's law

Point charges 1 nC and -1 nC are located at (0, 1m, 1m) and (1m, 1m, 0), respectively. Determine the force on a 5 nC charge located at the origin.

$$\vec{F}_{13} = \frac{K Q_1 Q_3}{|r_{13}|^3} \vec{r_{13}}$$

$$\vec{r_{13}} = \vec{r_3} - \vec{r_1} = -\hat{y} - \hat{z}$$

$$|\vec{r_{13}}| = \sqrt{-1^2 + -1^2} = \sqrt{2}$$

$$\vec{F_{13}} = \frac{9\times10^9 \times 1\times10^{-9} \times 5\times10^{-9}}{(\sqrt{2})^3}(-\hat{y} - \hat{z})$$

$$= -15.9\hat{y} - 15.9\hat{z} \ \ nN$$

$$\vec{F_{23}} = \frac{K Q_2 Q_3}{|r_{23}|^3} \vec{r_{23}}$$

$$\vec{r_{23}} = \vec{r_3} - \vec{r_2} = -\hat{x} - \hat{y}$$

$$|\vec{r_{23}}| = \sqrt{2}$$

$$\vec{F_{23}} = \frac{9\times10^9 \times -1\times10^{-9} \times 5\times10^{-9}}{(\sqrt{2})^3}(-\hat{x} - \hat{y})$$

$$= 15.9\hat{x} + 15.9\hat{y} \ \ nN$$

$$\therefore \vec{F_3} = \vec{F_{13}} + \vec{F_{23}} = 15.9\hat{x} - 15.9\hat{z} \ \ nN$$

Question 2: ***(35Mark)***

Objectives: This question is related to coordinate systems

Determine the total charge in the half sphere defined by

$0 \le r \le 4m$

$0 \le \theta \le \pi/2$

$0 \le \phi \le 2\pi$

if $\rho_v = 5r^2$ mC/m^3.

$$Q = \iiint \rho_v \, dv$$

$$dv = r^2 \sin\theta \, d\theta \, d\phi \, dr$$

$$Q = \int_{r=0}^{4} \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/2} 5r^2 r^2 \sin\theta \, d\theta \, d\phi \, dr$$

$$= 5 \frac{r^5}{5} \Big|_0^4 (-\cos\theta) \Big|_0^{\frac{\pi}{2}} \phi \Big|_0^{2\pi}$$

$$= 5 \times \frac{4^5}{5} \times -(0-1) \times 2\pi$$

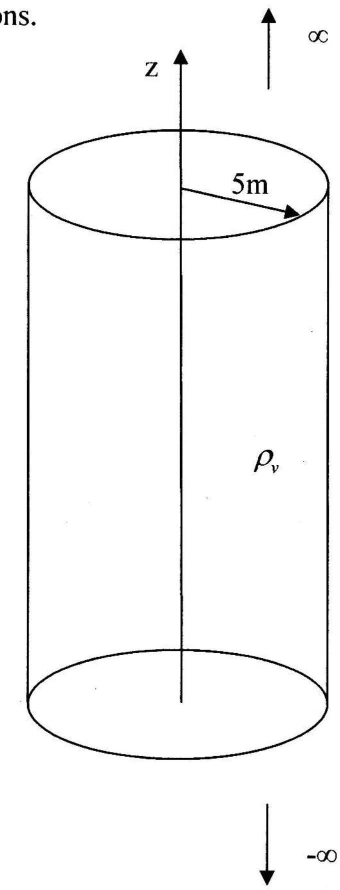
$$= 6434 \text{ mC}$$

$$= 6.434 \text{ C}$$

Question 3: ***(30Mark)***

Objectives: This question is related to Gauss's law

An infinitely long cylinder along the z axis of radius r=5m carries a uniform charge density ρ_v (C/m^3). Use Gauss's law to find $\vec{D}$ in all regions.



The problem is cylindrical so choose Gauss surface that is cylinder.

Due to symmetry $\vec{D}$ has only one component that is radially outward (in the $\hat{r}$ direction).

The space is divided into two regions:-

① $r \leq 5$

② $r > 5$

① $\underline{r \leq 5}$

$Q_{enc} = \pi h r^2 \rho_v$

$\phi = \oint \vec{D} \cdot \vec{ds}$

Note that on the top and bottom surfaces the flux density $\vec{D}$ is tangential to the surface $\Rightarrow$ $\vec{D} \cdot \vec{ds} = 0$

The only surface that the flux penetrates is the side surface

$\phi = D_r \, 2\pi r h$

Apply Gauss's law

$\phi = Q_{enc}$

$D_r \, 2\cancel{\pi}\cancel{r}\cancel{h} = \cancel{\pi}\cancel{h} r^{\cancel{2}} \rho_v \Rightarrow D_r = \frac{\rho_v r}{2} \ C/m^2$

② $\underline{r > 5}$

$Q_{enc} = \pi h 5^2 \times \rho_v = 25\pi h \rho_v$

$\phi = 2\pi r h \, D_r$

Apply Gauss's law

$\phi = Q_{enc}$

$2\cancel{\pi} r \cancel{h} \, D_r = 25 \cancel{\pi} \cancel{h} \rho_v$

$\Rightarrow D_r = \dfrac{12.5 \rho_v}{r} \quad C/m^2$